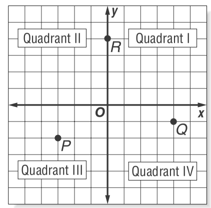
NAME ______________________ DATE ________ PERIOD _____

4-1 Study Guide and Intervention

The Coordinate Plane

Identify Points In the diagram at the right, points are located in reference to two perpendicular number lines called **axes**. The horizontal number line is the ***x*-axis**, and the vertical number line is the ***y*-axis**. The plane containing the *x*- and *y*-axes is called the **coordinate plane**. Points in the coordinate plane are named by ordered pairs of the form (x, y). The first number, or ***x*-coordinate** corresponds to a number on the *x*-axis. The second number, or ***y*-coordinate**, corresponds to a number on the *y*-axis.



The axes divide the coordinate plane into Quadrants I, II, III, and IV, as shown. The point where the axes intersect is called the **origin**. The origin has coordinates (0, 0).

Example 1 **Write an ordered pair for point *R* above.**

The *x*-coordinate is 0 and the *y*-coordinate is 4. Thus the ordered pair for *R* is (0, 4).

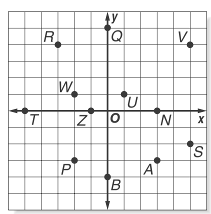
Example 2 **Write ordered pairs for points *P* and *Q* above. Then name the quadrant in which each point is located.**

The *x*-coordinate of *P* is −3 and the *y*-coordinate is −2. Thus the ordered pair for *P* is (−3, −2). *P* is in Quadrant III.

The *x*-coordinate of *Q* is 4 and the *y*-coordinate is −1. Thus the ordered pair for *Q* is (4, −1). *Q* is in Quadrant IV.

Exercises

Write the ordered pair for each point shown at the right. Name the quadrant in which the point is located.



1. *N* **(3, 0), none**
2. *P* **(−2, −3), III**
3. *Q* **(0, 5), none**
4. *R* **(−3, 4), II**
5. *S* **(5, −2), IV**
6. *T* **(−5, 0), none**
7. *U* **(1, 1), I**
8. *V* **(5, 4), I**
9. *W* **(−2, 1), II**
10. *Z* **(−1, 0), none**
11. *A* **(3, −3), IV**
12. *B* **(0, −4), none**
13. Write the ordered pair that describes a point 4 units down from and 3 units to the right of the origin. **(3, −4)**
14. Write the ordered pair that is 8 units to the left of the origin and lies on the *x*-axis. **(−8, 0)**

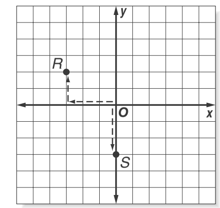
NAME ______________________ DATE ________ PERIOD _____

4-1 Study Guide and Intervention *(continued)*

The Coordinate Plane

Graph Points To **graph** an ordered pair means to draw a dot at the point on the coordinate plane that corresponds to the ordered pair. To graph an ordered pair (x, y), begin at the origin. Move left or right *x* units. From there, move up or down *y* units. Draw a dot at that point.

Example **Plot each point on a coordinate plane.**



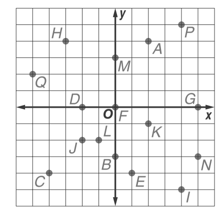
a. ***R*(−3, 2)**
Start at the origin. Move left 3 units since the *x*-coordinate is −3. Move up 2 units since the *y*-coordinate is 2. Draw a dot and label it *R*.

b. ***S*(0, −3)**
Start at the origin. Since the *x*-coordinate is 0, the point will be located on the *y*-axis. Move down 3 units since the *y*-coordinate is −3. Draw a dot and label it *S*.

Exercises

Plot each point on the coordinate plane at the right.

1–16. See graph.



1. *A*(2, 4)
2. *B*(0, −3)
3. *C*(−4, −4)
4. *D*(−2, 0)
5. *E*(1, −4)
6. *F*(0, 0)
7. *G*(5, 0)
8. *H*(−3, 4)
9. *I*(4, −5)
10. *J*(−2, −2)
11. *K*(2, −1)
12. *L*(−1, −2)
13. *M*(0, 3)
14. *N*(5, −3)
15. *P*(4, 5)
16. *Q*(−5, 2)

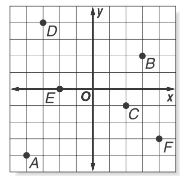
Lesson 4-1

NAME ______________________ DATE ________ PERIOD ____

4-1 Skills Practice

The Coordinate Plane

Write the ordered pair for each point shown at the right. Name the quadrant in which the point is located.

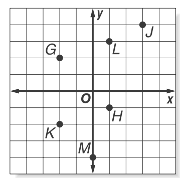


1. A **(−4, −4); III** **2.** B **(3, 2); I**

3. C **(2, −1); IV** **4.** D **(−3, 4); II**

5. E **(−2, 0); none** **6.** F **(4, −3); IV**

Write the ordered pair for each point shown at the right. Name the quadrant in which the point is located.

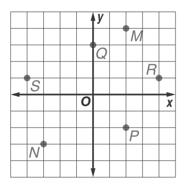


7. G **(−2, 2); II** **8.** H **(1, −1); IV**

9. J **(3, 4); I** **10.** K **(−2, −2); III**

11. L **(1, 3); I** **12.** M **(0, −4); none**

Plot each point on the coordinate plane at the right.

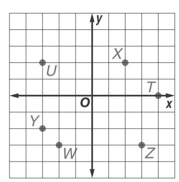


13. $M(2, 4)$ **14.** $N(-3, -3)$

15. $P(2, -2)$ **16.** $Q(0, 3)$

17. $R(4, 1)$ **18.** $S(-4, 1)$

Plot each point on the coordinate plane at the right.



19. $T(4, 0)$ **20.** $U(-3, 2)$

21. $W(-2, -3)$ **22.** $X(2, 2)$

23. $Y(-3, -2)$ **24.** $Z(3, -3)$

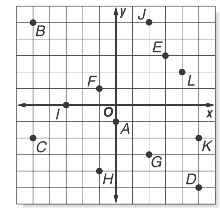
Lesson 4-1

NAME ______________________ DATE ________ PERIOD ____

4-1 Practice (Average)

The Coordinate Plane

Write the ordered pair for each point shown at the right. Name the quadrant in which the point is located.



1. A **(0, −1); none** **2.** B **(−5, 5); II**

3. C **(−5, −2); III** **4.** D **(5, −5); IV**

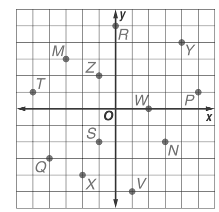
5. E **(3, 3); I** **6.** F **(−1, 1); II**

7. G **(2, −3); IV** **8.** H **(−1, −4); III**

9. I **(−3, 0); none** **10.** J **(2, 5); I**

11. K **(5, −2); IV** **12.** L **(4, 2); I**

Plot each point on the coordinate plane at the right.



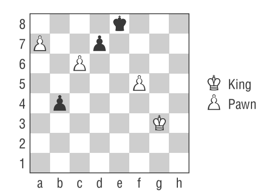
13. $M(-3, 3)$ **14.** $N(3, -2)$ **15.** $P(5, 1)$

16. $Q(-4, -3)$ **17.** $R(0, 5)$ **18.** $S(-1, -2)$

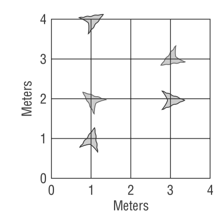
19. $T(-5, 1)$ **20.** $V(1, -5)$ **21.** $W(2, 0)$

22. $X(-2, -4)$ **23.** $Y(4, 4)$ **24.** $Z(-1, 2)$

25. CHESS Letters and numbers are used to show the positions of chess pieces and to describe their moves. For example, in the diagram at the right, a white pawn is located at f5. Name the positions of each of the remaining chess pieces.
white pawns: a7, c6; black pawns: b4, d7; white king: g3; black king: e8



ARCHAEOLOGY For Exercises 26 and 27, use the grid at the right that shows the location of arrowheads excavated at a *midden*—a place where people in the past dumped trash, food remains, and other discarded items.



26. Write the coordinates of each arrowhead. **(1, 1), (3, 2), (1, 4)**

27. Suppose an archaeologist discovers two other arrowheads located at (1, 2) and (3, 3). Draw an arrowhead at each of these locations on the grid.

Answers

NAME ______________ DATE ______ PERIOD ____

4-1 Reading to Learn Mathematics

The Coordinate Plane

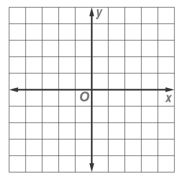
Pre-Activity **How do archaeologists use coordinate systems?**

Read the introduction to Lesson 4-1 at the top of page 192 in your textbook.

What do the terms *grid system*, *grid*, and *coordinate system* mean to you?
See students' work.

Reading the Lesson

1. Use the coordinate plane shown at the right.



a. Label the origin *O*.

b. Label the *y*-axis *y*.

c. Label the *x*-axis *x*.

2. Explain why the coordinates of the origin are (0, 0).
Sample answer: The origin is the intersection of two number lines at their common zero point.

3. Use the ordered pair (−2, 3).

a. Explain how to identify the *x*- and *y*-coordinates. **The *x*-coordinate is the first number, the *y*-coordinate is the second number.**

b. Name the *x*- and *y*-coordinates. **The *x*-coordinate is −2, the *y*-coordinate is 3.**

c. Describe the steps you would use to locate the point for (−2, 3) on the coordinate plane. **Start at the origin. Move left 2 units. Then move up 3 units.**

4. What does the term *quadrant* mean? **Sample answer: one of the four regions in the coordinate plane**

Helping You Remember

5. Explain how the way the axes are labeled on the coordinate plane can help you remember how to plot the point for an ordered pair. **Sample answer: The right side of the horizontal axis is labeled with the letter *x*. This is the side of the horizontal number line where you find positive numbers. The top end of the vertical number line is labeled with the letter *y*. This is the part of the vertical number line where you find positive numbers.**

Lesson 4-1

NAME ______________ DATE ______ PERIOD ____

4-1 Enrichment

Midpoint

The *midpoint* of a line segment is the point that lies exactly halfway between the two endpoints of the segment. The coordinates of the midpoint of a line segment whose endpoints are (x_1, y_1) and (x_2, y_2) are given by $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$.

Find the midpoint of each line segment with the given endpoints.

1. (7, 1) and (−3, 1)
(2, 1)

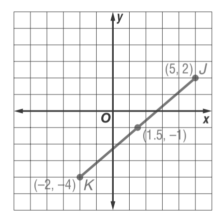
2. (5, −2) and (9, −8)
(7, −5)

3. (−4, 4) and (4, −4)
(0, 0)

4. (−3, −6) and (−10, −15)
(−6.5, −10.5)

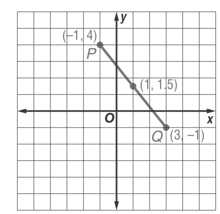
Plot each segment in the coordinate plane. Then find the coordinates of the midpoint.

5. $\overline{JK}$ with $J(5, 2)$ and $K(-2, -4)$



(1.5, −1)

6. $\overline{PQ}$ with $P(-1, 4)$ and $Q(3, -1)$



(1, 1.5)

You are given the coordinates of one endpoint of a line segment and the midpoint *M*. Find the coordinates of the other endpoint.

7. $A(-10, 3)$ and $M(-6, 7)$
(−2, 11)

8. $D(-1, 4)$ and $M(3, -6)$
(7, −16)

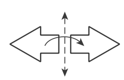
NAME ______________________ DATE __________ PERIOD _____

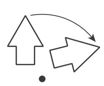
4-2 Study Guide and Intervention
Transformations on the Coordinate Plane

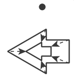
Transform Figures **Transformations** are movements of geometric figures. The **preimage** is the position of the figure before the transformation, and the **image** is the position of the figure after the transformation.

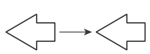
Reflection	A figure is flipped over a line.
Translation	A figure is slid horizontally, vertically, or both.
Dilation	A figure is enlarged or reduced.
Rotation	A figure is turned around a point.

Example **Determine whether each transformation is a *reflection*, *translation*, *dilation*, or *rotation*.**

a. 
The figure has been flipped over a line, so this is a reflection.

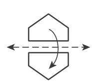
b. 
The figure has been turned around a point, so this is a rotation.

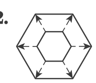
c. 
The figure has been reduced in size, so this is a dilation.

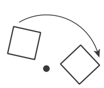
d. 
The figure has been shifted horizontally to the right, so this is a translation.

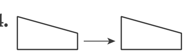
Exercises

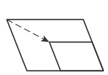
Determine whether each transformation is a *reflection*, *translation*, *dilation*, or *rotation*.

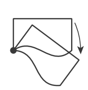
1. 
reflection

2. 
dilation

3. 
rotation

4. 
translation

5. 
dilation

6. 
rotation

Lesson 4-2

NAME ______________________ DATE __________ PERIOD _____

4-2 Study Guide and Intervention *(continued)*
Transformations on the Coordinate Plane

Transform Figures on the Coordinate Plane You can perform transformations on a coordinate plane by changing the coordinates of each vertex. The vertices of the image of the transformed figure are indicated by the symbol ′, which is read *prime*.

Reflection over *x*-axis	$(x, y) \rightarrow (x, -y)$
Reflection over *y*-axis	$(x, y) \rightarrow (-x, y)$
Translation	$(x, y) \rightarrow (x + a, y + b)$
Dilation	$(x, y) \rightarrow (kx, ky)$
Rotation 90° counterclockwise	$(x, y) \rightarrow (-y, x)$
Rotation 180°	$(x, y) \rightarrow (-x, -y)$

Example **A triangle has vertices $A(-1, 1)$, $B(2, 4)$, and $C(3, 0)$. Find the coordinates of the vertices of each image below.**

a. reflection over the x-axis

To reflect a point over the x-axis, multiply the y-coordinate by -1.

$A(-1, 1) \rightarrow A'(-1, -1)$
$B(2, 4) \rightarrow B'(2, -4)$
$C(3, 0) \rightarrow C'(3, 0)$

The coordinates of the image vertices are $A'(-1, -1)$, $B'(2, -4)$, and $C'(3, 0)$.

b. dilation with a scale factor of 2

Find the coordinates of the dilated figure by multiplying the coordinates by 2.

$A(-1, 1) \rightarrow A'(-2, 2)$
$B(2, 4) \rightarrow B'(4, 8)$
$C(3, 0) \rightarrow C'(6, 0)$

The coordinates of the image vertices are $A'(-2, 2)$, $B'(4, 8)$, and $C'(6, 0)$.

Exercises

Find the coordinates of the vertices of each figure after the given transformation is performed.

1. triangle RST with $R(-2, 4)$, $S(2, 0)$, $T(-1, -1)$ reflected over the y-axis
$R'(2, 4)$, $S'(-2, 0)$, $T'(1, -1)$

2. triangle ABC with $A(0, 0)$, $B(2, 4)$, $C(3, 0)$ rotated about the origin 180°
$A'(0, 0)$, $B'(-2, -4)$, $C'(-3, 0)$

3. parallelogram $ABCD$ with $A(-3, 0)$, $B(-2, 3)$, $C(3, 3)$, $D(2, 0)$ translated 3 units down
$A'(-3, -3)$, $B'(-2, 0)$, $C'(3, 0)$, $D'(2, -3)$

4. quadrilateral $RSTU$ with $R(-2, 2)$, $S(2, 4)$, $T(4, 4)$, $U(4, 0)$ dilated by a factor of $\frac{1}{2}$
$R'(-1, 1)$, $S'(1, 2)$, $T'(2, 2)$, $U'(2, 0)$

5. triangle ABC with $A(-4, 0)$, $B(-2, 3)$, $C(0, 0)$ rotated counterclockwise 90°
$A'(0, -4)$, $B'(-3, -2)$, $C'(0, 0)$

6. hexagon $ABCDEF$ with $A(0, 0)$, $B(-2, 3)$, $C(0, 4)$, $D(3, 4)$, $E(4, 2)$, $F(3, 0)$ translated 2 units up and 1 unit to the left
$A'(-1, 2)$, $B'(-3, 5)$, $C'(-1, 6)$, $D'(2, 6)$, $E'(3, 4)$, $F'(2, 2)$

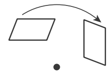
Answers

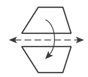
NAME ______________________ DATE __________ PERIOD _____

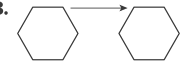
4-2 Skills Practice

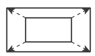
Transformations on the Coordinate Plane

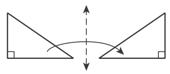
Identify each transformation as a *reflection*, *translation*, *dilation*, or *rotation*.

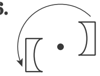
1. 
rotation

2. 
reflection

3. 
translation

4. 
dilation

5. 
reflection

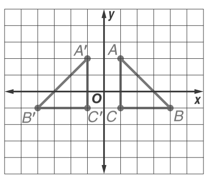
6. 
rotation

For Exercises 7–10, complete parts a and b.

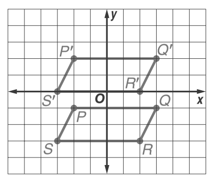
a. Find the coordinates of the vertices of each figure after the given transformation is performed.

b. Graph the preimage and its image.

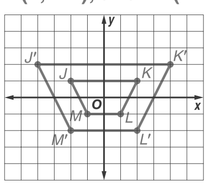
7. triangle ABC with $A(1, 2)$, $B(4, -1)$, and $C(1, -1)$ reflected over the y-axis
$A'(-1, 2)$, $B'(-4, -1)$, and $C'(-1, -1)$



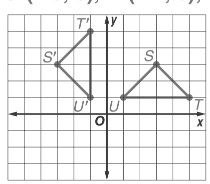
8. parallelogram $PQRS$ with $P(-2, -1)$, $Q(3, -1)$, $R(2, -3)$, and $S(-3, -3)$ translated 3 units up **$P'(-2, 2)$, $Q'(3, 2)$, $R'(2, 0)$, and $S'(-3, 0)$**



9. trapezoid $JKLM$ with $J(-2, 1)$, $K(2, 1)$, $L(1, -1)$, and $M(-1, -1)$ dilated by a scale factor of 2 **$J'(-4, 2)$, $K'(4, 2)$, $L'(2, -2)$, and $M'(-2, -2)$**



10. triangle STU with $S(3, 3)$, $T(5, 1)$, and $U(1, 1)$ rotated 90° counterclockwise about the origin
$S'(-3, 3)$, $T'(-1, 5)$, $U'(-1, 1)$



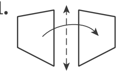
Lesson 4-2

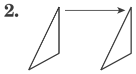
NAME ______________________ DATE __________ PERIOD _____

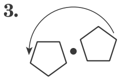
4-2 Practice (Average)

Transformations on the Coordinate Plane

Identify each transformation as a *reflection*, *translation*, *dilation*, or *rotation*.

1. 
reflection

2. 
translation

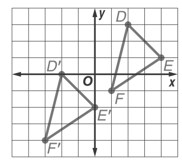
3. 
rotation

For Exercises 4–6, complete parts a and b.

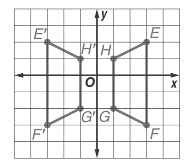
a. Find the coordinates of the vertices of each figure after the given transformation is performed.

b. Graph the preimage and its image.

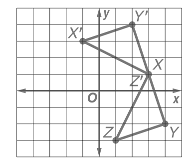
4. triangle DEF with $D(2, 3)$, $E(4, 1)$, and $F(1, -1)$ translated 4 units left and 3 units down
$D'(-2, 0)$, $E'(0, -2)$, $F'(-3, -4)$



5. trapezoid $EFGH$ with $E(3, 2)$, $F(3, -3)$, $G(1, -2)$, and $H(1, 1)$ reflected over the y-axis
$E'(-3, 2)$, $F'(-3, -3)$, $G'(-1, -2)$, $H'(-1, 1)$

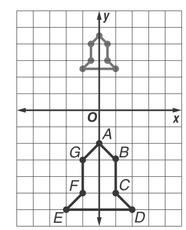


6. triangle XYZ with $X(3, 1)$, $Y(4, -2)$, and $Z(1, -3)$ rotated 90° counterclockwise about the origin
$X'(-1, 3)$, $Y'(2, 4)$, $Z'(3, 1)$



GRAPHICS For Exercises 7–9, use the diagram at the right and the following information.

A designer wants to dilate the rocket by a scale factor of $\frac{1}{2}$, and then translate it $5\frac{1}{2}$ units up.

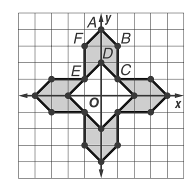


7. Write the coordinates for the vertices of the rocket.
$A(0, -2)$, $B(1, -3)$, $C(1, -5)$, $D(2, -6)$, $E(-2, -6)$, $F(-1, -5)$, $G(-1, -3)$

8. Find the coordinates of the final position of the rocket.
$A'\left(0, 4\frac{1}{2}\right)$, $B'\left(\frac{1}{2}, 4\right)$, $C'\left(\frac{1}{2}, 3\right)$, $D'\left(1, 2\frac{1}{2}\right)$, $E'\left(-1, 2\frac{1}{2}\right)$, $F'\left(-\frac{1}{2}, 3\right)$, $G'\left(-\frac{1}{2}, 4\right)$

9. Graph the image on the coordinate plane.

10. **DESIGN** Ramona transformed figure $ABCDEF$ to design a pattern for a quilt. Name two different sets of transformations she could have used to design the pattern. **Sample answer: reflection over the x-axis, 90° counterclockwise rotation, and then reflection over the y-axis; three 90° counterclockwise rotations**



NAME ______________________ DATE ________ PERIOD _____

4-2 Reading to Learn Mathematics

Transformations on the Coordinate Plane

Pre-Activity **How are transformations used in computer graphics?**

Read the introduction to Lesson 4-2 at the top of page 197 in your textbook.

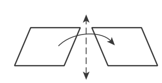
In the sentence, "Computer graphic designers can create movement that mimics real-life situations," what phrase indicates the use of transformations? **create movement**

Reading the Lesson

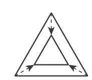
1. Suppose you look at a diagram that shows two figures *ABCDE* and *A'B'C'D'E'*. If one figure was obtained from the other by using a transformation, how do you tell which was the original figure? **The letters that have no prime symbols are used for vertices of the original figure.**

2. Write the letter of the term and the Roman numeral of the figure that best matches each statement.

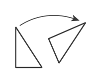
a. A figure is flipped over a line. **C, I** — **A.** dilation — **I.**



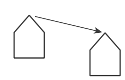
b. A figure is turned around a point. **D, III** — **B.** translation — **II.**



c. A figure is enlarged or reduced. **A, II** — **C.** reflection — **III.**



d. A figure is slid horizontally, vertically, or both. **B, IV** — **D.** rotation — **IV.**



Helping You Remember

3. Give examples of things in everyday life that can help you remember what reflections, dilations, and rotations are. **Sample answer: For a reflection, think of looking at yourself in a mirror. For a dilation, think of how your hand looks if you hold it far from your face and then move it straight in, very close to your face. For a rotation, think of twisting the top of a jar to open the jar.**

NAME ______________________ DATE ________ PERIOD _____

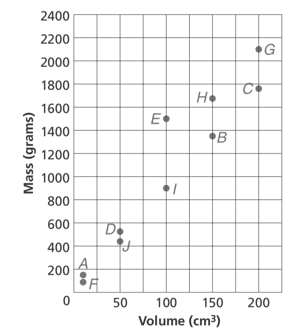
4-2 Enrichment

The Legendary City of Ur

The city of Ur was founded more than five thousand years ago in Mesopotamia (modern-day Iraq). It was one of the world's first cities. Between 1922 and 1934, archeologists discovered many treasures from this ancient city. A large cemetery from the 26th century B.C. was found to contain large quantities of gold, silver, bronze, and jewels. The many cultural artifacts that were found, such as musical instruments, weapons, mosaics, and statues, have provided historians with valuable clues about the civilization that existed in early Mesopotamia.

1. Suppose that the ordered pairs below represent the volume (cm^3) and mass (grams) of ten artifacts from the city of Ur. Plot each point on the graph.

A(10, 150)
B(150, 1350)
C(200, 1760)
D(50, 525)
E(100, 1500)
F(10, 88)
G(200, 2100)
H(150, 1675)
I(100, 900)
J(50, 440)



2. The equation relating mass, density, and volume for silver is $m = 10.5V$. Which of the points in Exercise 1 are solutions for this equation?
D* and *G

3. Suppose that the equation $m = 8.8V$ relates mass, density, and volume for the kind of bronze used in the ancient city of Ur. Which of the points in Exercise 1 are solutions for this equation?
C*, *F*, *J

4. Explain why the graph in Exercise 1 shows only quadrant 1.
Neither the mass nor the volume can be a negative number.

NAME ______________________ DATE ____________ PERIOD _____

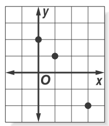
4-3 Study Guide and Intervention

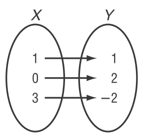
Relations

Represent Relations A **relation** is a set of ordered pairs. A relation can be represented by a set of ordered pairs, a table, a graph, or a **mapping**. A mapping illustrates how each element of the domain is paired with an element in the range.

Example 1 **Express the relation {(1, 1), (0, 2), (3, −2)} as a table, a graph, and a mapping. State the domain and range of the relation.**

x	y
1	1
0	2
3	−2





The domain for this relation is {0, 1, 3}.
The range for this relation is {−2, 1, 2}.

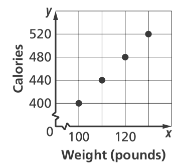
Example 2 **A person playing racquetball uses 4 calories per hour for every pound he or she weighs.**

a. Make a table to show the relation between weight and calories burned in one hour for people weighing 100, 110, 120, and 130 pounds.
Source: *The Math Teacher's Book of Lists*

x	y
100	400
110	440
120	480
130	520

b. Give the domain and range.
domain: {100, 110, 120, 130}
range: {400, 440, 480, 520}

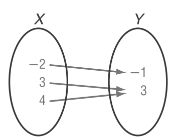
c. Graph the relation.

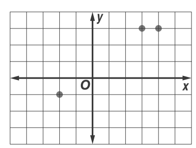


Exercises

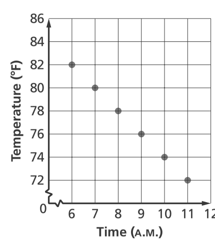
1. Express the relation {(−2, −1), (3, 3), (4, 3)} as a table, a graph, and a mapping. Then determine the domain and range.
domain: {−2, 3, 4}; range: {−1, 3}

x	y
−2	**−1**
3	**3**
4	**3**





2. The temperature in a house drops 2° for every hour the air conditioner is on between the hours of 6 A.M. and 11 A.M. Make a graph to show the relationship between time and temperature if the temperature at 6 A.M. was 82°F.



Lesson 4-3

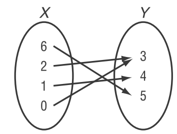
NAME ______________________ DATE ____________ PERIOD _____

4-3 Study Guide and Intervention *(continued)*

Relations

Inverse Relations The **inverse** of any relation is obtained by switching the coordinates in each ordered pair.

Example **Express the relation shown in the mapping as a set of ordered pairs. Then write the inverse of the relation.**



Relation: {(6, 5), (2, 3), (1, 4), (0, 3)}
Inverse: {(5, 6), (3, 2), (4, 1), (3, 0)}

Exercises

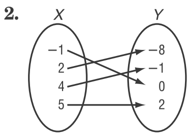
Express the relation shown in each table, mapping, or graph as a set of ordered pairs. Then write the inverse of each relation.

1.

x	y
−2	4
−1	3
2	1
4	5

{(−2, 4), (−1, 3), (2, 1), (4, 5)}; {(4, −2), (3, −1), (1, 2), (5, 4)}

2.



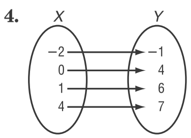
{(−1, 0), (2, −8), (4, −1), (5, 2)}; {(0, −1), (−8, 2), (−1, 4), (2, 5)}

3.

x	y
−3	5
−2	−1
1	0
2	4

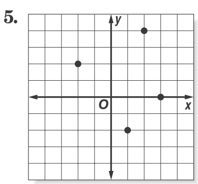
{(−3, 5), (−2, −1), (1, 0), (2, 4)}; {(5, −3), (−1, −2), (0, 1), (4, 2)}

4.



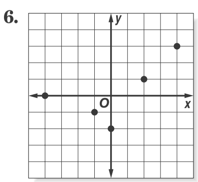
{(−2, −1), (0, 4), (1, 6), (4, 7)}; {(−1, −2), (4, 0), (6, 1), (7, 4)}

5.



{(−2, 2), (1, −2), (2, 4), (3, 0)}; {(2, −2), (−2, 1), (4, 2), (0, 3)}

6.



{(−4, 0), (−1, −1), (0, −2), (2, 1), (4, 3)}; {(0, −4), (−1, −1), (−2, 0), (1, 2), (3, 4)}

NAME ______________________ DATE ___________ PERIOD _____

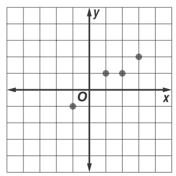
4-3 Skills Practice

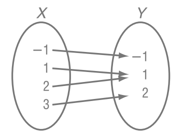
Relations

Express each relation as a table, a graph, and a mapping. Then determine the domain and range.

1. {(−1, −1), (1, 1), (2, 1), (3, 2)}

x	y
−1	−1
1	1
2	1
3	2

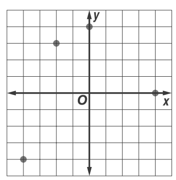


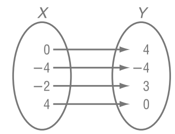


D = {−1, 1, 2, 3}; R = {−1, 1, 2}

2. {(0, 4), (−4, −4), (−2, 3), (4, 0)}

x	y
0	4
−4	−4
−2	3
4	0

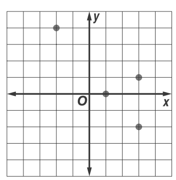


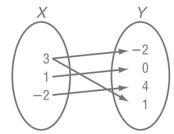


D = {−4, −2, 0, 4}; R = {−4, 0, 3, 4}

3. {(3, −2), (1, 0), (−2, 4), (3, 1)}

x	y
3	−2
1	0
−2	4
3	1





D = {−2, 1, 3}; R = {−2, 0, 1, 4}

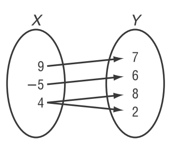
Express the relation shown in each table, mapping, or graph as a set of ordered pairs. Then write the inverse of the relation.

4.

x	y
3	−5
−4	3
7	6
1	−2

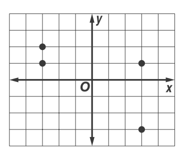
{(3, −5), (−4, 3), (7, 6), (1, −2)}; {(−5, 3), (3, −4), (6, 7), (−2, 1)}

5.



{(9, 7), (−5, 6), (4, 8), (4, 2)}; {(7, 9), (6, −5), (8, 4), (2, 4)}

6.



{(−3, 1), (−3, 2), (3, 1), (3, −3)}; {(1, −3), (2, −3), (1, 3), (−3, 3)}

NAME ______________________ DATE ___________ PERIOD _____

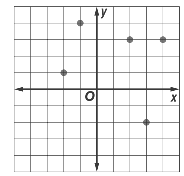
4-3 Practice (Average)

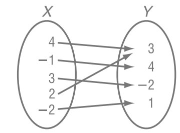
Relations

Express each relation as a table, a graph, and a mapping. Then determine the domain and range.

1. {(4, 3), (−1, 4), (3, −2), (2, 3), (−2, 1)}

x	y
4	3
−1	4
3	−2
2	3
−2	1





D = {−2, −1, 2, 3, 4}; R = {−2, 1, 3, 4}

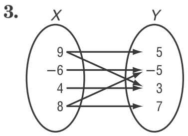
Express the relation shown in each table, mapping, or graph as a set of ordered pairs. Then write the inverse of the relation.

2.

x	y
0	9
−8	3
2	−6
1	4

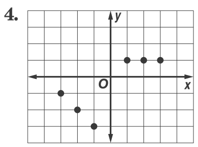
{(0, 9), (−8, 3), (2, −6), (1, 4)}; {(9, 0), (3, −8), (−6, 2), (4, 1)}

3.



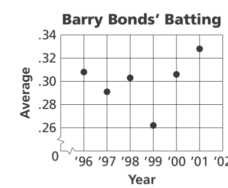
{(9, 5), (9, 3), (−6, −5), (4, 3), (8, −5), (8, 7)}; {(5, 9), (3, 9), (−5, −6), (3, 4), (−5, 8), (7, 8)}

4.



{(−3, −1), (−2, −2), (−1, −3), (1, 1), (2, 1), (3, 1)}; {(−1, −3), (−2, −2), (−3, −1), (1, 1), (1, 2), (1, 3)}

BASEBALL For Exercises 5 and 6, use the graph that shows the batting average for Barry Bonds of the San Francisco Giants. Source: www.sportsillustrated.cnn.com



5. Find the domain and estimate the range.
D = {1996, 1997, 1998, 1999, 2000, 2001};
R = {.262, .291, .303, .306, .308, .328}

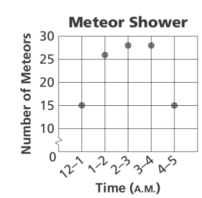
6. Which seasons did Bonds have the lowest and highest batting averages? lowest: 1999, highest: 2001

METEORS For Exercises 7 and 8, use the table that shows the number of meteors Ann observed each hour during a meteor shower.

Time (A.M.)	Number of Meteors
12	15
1	26
2	28
3	28
4	15

7. What are the domain and range?
D = {12, 1, 2, 3, 4}; R = {15, 26, 28}

8. Graph the relation.



Lesson 4-3

Answers

NAME ______________________ DATE __________ PERIOD _____

4-3 Reading to Learn Mathematics

Relations

Pre-Activity **How can relations be used to represent baseball statistics?**

Read the introduction to Lesson 4-3 at the top of page 205 in your textbook.

In 1997, Ken Griffey, Jr. had __56__ home runs and __121__ strikeouts.

This can be represented with the ordered pair (__56__, __121__).

Reading the Lesson

1. Look at page 205 in your textbook. There you see the same relation represented by a set of ordered pairs, a table, a graph, and a mapping.

   a. In the list of ordered pairs, where do you see the numbers for the domain? the numbers for the range? **before the commas; after the commas**

   b. What parts of the table show the domain and the range? **The column of numbers under the letter *x* shows the domain, and the column of numbers under the letter *y* shows the range.**

   c. How do the table, the graph, and the mapping show that there are three ordered pairs in the relation? **The table has three rows of numbers, the graph shows three points marked with dots, and the mapping uses three arrows.**

2. Which tells you more about a relation, a list of the ordered pairs in the relation or the domain and range of the relation? Explain. **Sample answer: A list of the ordered pairs tells you more. You can use it to find the domain and the range, and you know exactly how the numbers are paired. If you only know the domain and the range, you cannot be sure how the numbers are paired.**

3. Describe how you would find the inverse of the relation {(1, 2), (2, 4), (3, 6), (4, 8)}. **Switch the coordinates in each ordered pair to get {(2, 1), (4, 2), (6, 3), (8, 4)}.**

Helping You Remember

4. The first letters in two words and their order in the alphabet can sometimes help you remember their mathematical meaning. Two key terms in this lesson are *domain* and *range*. Describe how the alphabet method could help you remember their meaning. **Sample answer: *d* comes first for the first coordinate, *r* comes second for the second coordinate.**

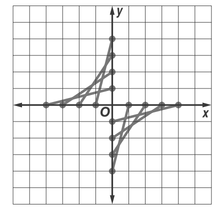
Lesson 4-3

NAME ______________________ DATE __________ PERIOD _____

4-3 Enrichment

Inverse Relations

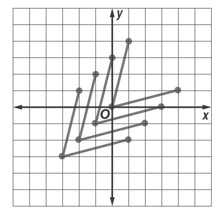
On each grid below, plot the points in Sets A and B. Then connect the points in Set A with the corresponding points in Set B. Then find the inverses of Set A and Set B, plot the two sets, and connect those points.



Set A	Set B
(−4, 0)	(0, 1)
(−3, 0)	(0, 2)
(−2, 0)	(0, 3)
(−1, 0)	(0, 4)

Inverse

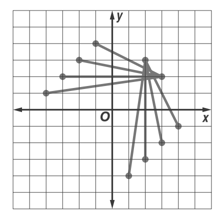
	Set A	Set B
1.	(0, −4)	(1, 0)
2.	(0, −3)	(2, 0)
3.	(0, −2)	(3, 0)
4.	(0, −1)	(4, 0)



Set A	Set B
(−3, −3)	(−2, 1)
(−2, −2)	(−1, 2)
(−1, −1)	(0, 3)
(0, 0)	(1, 4)

Inverse

	Set A	Set B
5.	(−3, −3)	(1, −2)
6.	(−2, −2)	(2, −1)
7.	(−1, −1)	(3, 0)
8.	(0, 0)	(4, 1)



Set A	Set B
(−4, 1)	(3, 2)
(−3, 2)	(3, 2)
(−2, 3)	(3, 2)
(−1, 4)	(3, 2)

Inverse

	Set A	Set B
9.	(1, −4)	(2, 3)
10.	(2, −3)	(2, 3)
11.	(3, −2)	(2, 3)
12.	(4, −1)	(2, 3)

13. What is the graphical relationship between the line segments you drew connecting points in Sets A and B and the line segments connecting points in the inverses of those two sets?
Answers will vary. A possible answer is that the graphs are reflected across the line $y = x$ by their inverses.

NAME ______________________ DATE ________ PERIOD _____

4-4 Study Guide and Intervention

Equations as Relations

Solve Equations The equation $y = 3x - 4$ is an example of an **equation in two variables** because it contains two variables, x and y. The **solution** of an equation in two variables is an ordered pair of replacements for the variables that results in a true statement when substituted into the equation.

Example 1 **Find the solution set for $y = -2x - 1$, given the replacement set $\{(-2, 3), (0, -1), (1, -2), (3, 1)\}$.**

Make a table. Substitute the x and y-values of each ordered pair into the equation.

x	y	$y = -2x - 1$	True or False
−2	3	$3 = -2(-2) - 1$ $3 = 3$	True
0	−1	$-1 = -2(0) - 1$ $-1 = -1$	True
1	−2	$-2 = -2(1) - 1$ $-2 = -3$	False
3	1	$1 = -2(3) - 1$ $1 = -7$	False

The ordered pairs $(-2, 3)$, and $(0, -1)$ result in true statements. The solution set is $\{(-2, 3), (0, -1)\}$.

Example 2 **Solve $b = 2a - 1$ if the domain is $\{-2, -1, 0, 2, 4\}$.**

Make a table. The values of a come from the domain. Substitute each value of a into the equation to determine the corresponding values of b in the range.

a	$2a - 1$	b	(a, b)
−2	2(−2) − 1	−5	(−2, −5)
−1	2(−1) − 1	−3	(−1, −3)
0	2(0) − 1	−1	(0, −1)
2	2(2) − 1	3	(2, 3)
4	2(4) − 1	7	(4, 7)

The solution set is $\{(-2, -5), (-1, -3), (0, -1), (2, 3), (4, 7)\}$.

Exercises

Find the solution set of each equation, given the replacement set.

1. $y = 3x + 1$; $\{(0, 1), \left(\frac{1}{3}, 2\right), \left(-1, -\frac{2}{3}\right), (-1, -2)\}$ **$\{(0, 1), \left(\frac{1}{3}, 2\right), (-1, -2)\}$**

2. $3x - 2y = 6$; $\{(-2, 3), (0, 1), (0, -3), (2, 0)\}$ **$\{(0, -3), (2, 0)\}$**

3. $2x = 5 - y$; $\{(1, 3), (2, 1), (3, 2), (4, 3)\}$ **$\{(1, 3), (2, 1)\}$**

Solve each equation if the domain is $\{-4, -2, 0, 2, 4\}$.

4. $x + y = 4$ **$\{(-4, 8), (-2, 6), (0, 4), (2, 2)\ (4, 0)\}$**

5. $y = -4x - 6$ **$\{(-4, 10), (-2, 2), (0, -6), (2, -14), (4, -22)\}$**

6. $5a - 2b = 10$ **$\{(-4, -15), (-2, -10), (0, -5), (2, 0), (4, 5)\}$**

7. $3x - 2y = 12$ **$\{(-4, -12), (-2, -9), (0, -6), (2, -3)\ (4, 0)\}$**

8. $6x + 3y = 18$ **$\{(-4, 14), (-2, 10), (0, 6), (2, 2), (4, -2)\}$**

9. $4x + 8 = 2y$ **$\{(-4, -4), (-2, 0), (0, 4), (2, 8), (4, 12)\}$**

10. $x - y = 8$ **$\{(-4, -12), (-2, -10), (0, -8), (2, -6)\ (4, -4)\}$**

11. $2x + y = 10$ **$\{(-4, 18), (-2, 14), (0, 10), (2, 6), (4, 2)\}$**

NAME ______________________ DATE ________ PERIOD _____

4-4 Study Guide and Intervention *(continued)*

Equations as Relations

Graph Solution Sets You can graph the ordered pairs in the solution set of an equation in two variables. The domain contains values represented by the **independent variable**. The range contains the corresponding values represented by the **dependent variable**, which are determined by the given equation.

Example **Solve $4x + 2y = 12$ if the domain is $\{-1, 0, 2, 4\}$. Graph the solution set.**

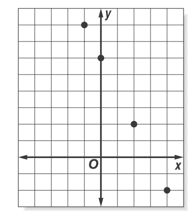
First solve the equation for y in terms of x.

$$4x + 2y = 12 \quad \text{Original equation}$$
$$4x + 2y - 4x = 12 - 4x \quad \text{Subtract } 4x \text{ from each side.}$$
$$2y = 12 - 4x \quad \text{Simplify.}$$
$$\frac{2y}{2} = \frac{12 - 4x}{2} \quad \text{Divide each side by 2.}$$
$$y = 6 - 2x \quad \text{Simplify.}$$

Substitute each value of x from the domain to determine the corresponding value of y in the range.

x	$6 - 2x$	y	(x, y)
−1	6 − 2(−1)	8	(−1, 8)
0	6 − 2(0)	6	(0, 6)
2	6 − 2(2)	2	(2, 2)
4	6 − 2(4)	−2	(4, −2)

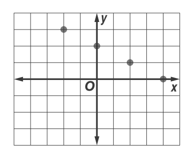
Graph the solution set.



Exercises

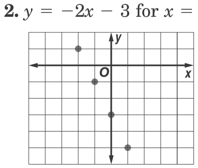
Solve each equation for the given domain. Graph the solution set.

1. $x + 2y = 4$ for $x = \{-2, 0, 2, 4\}$



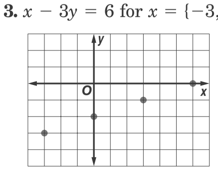
$\{(-2, 3), (0, 2), (2, 1), (4, 0)\}$

2. $y = -2x - 3$ for $x = \{-2, -1, 0, 1\}$



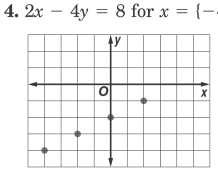
$\{(-2, 1), (-1, -1), (0, -3), (1, -5)\}$

3. $x - 3y = 6$ for $x = \{-3, 0, 3, 6\}$



$\{(-3, -3), (0, -2), (3, -1), (6, 0)\}$

4. $2x - 4y = 8$ for $x = \{-4, -2, 0, 2\}$



$\{(-4, -4), (-2, -3), (0, -2), (2, -1)\}$

Lesson 4-4

Answers

NAME ______________________ DATE ________ PERIOD _____

4-4 Skills Practice

Equations as Relations

Find the solution set for each equation, given the replacement set.

1. $y = 3x - 1$; {(2, 5), (−2, 7), (0, −1), (1, 1)} **{(2, 5), (0, −1)}**

2. $y = 2x + 4$; {(−1, −2), (−3, 2), (1, 6), (−2, 8)} **{(1, 6)}**

3. $y = 7 - 2x$; {(3, 1), (4, −1), (5, −3), (−1, 5)} **{(3, 1), (4, −1), (5, −3)}**

4. $-3x + y = 2$; {(−3, 7), (−2, −4), (−1, −1), (3, 11)} **{(−2, −4), (−1, −1), (3, 11)}**

Solve each equation if the domain is {−2, −1, 0, 2, 5}.

5. $y = x + 4$ **{(−2, 2), (−1, 3), (0, 4), (2, 6), (5, 9)}**

6. $y = 3x - 2$ **{(−2, −8), (−1, −5), (0, −2), (2, 4), (5, 13)}**

7. $y = 2x + 1$ **{(−2, −3), (−1, −1), (0, 1), (2, 5), (5, 11)}**

8. $x = y + 2$ **{(−2, −4), (−1, −3), (0, −2), (2, 0), (5, 3)}**

9. $x = 3 - y$ **{(−2, 5), (−1, 4), (0, 3), (2, 1), (5, −2)}**

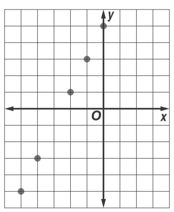
10. $2x + y = 4$ **{(−2, 8), (−1, 6), (0, 4), (2, 0), (5, −6)}**

11. $2x - y = 7$ **{(−2, −11), (−1, −9), (0, −7), (2, −3), (5, 3)}**

12. $4x + 2y = 6$ **{(−2, 7), (−1, 5), (0, 3), (2, −1), (5, −7)}**

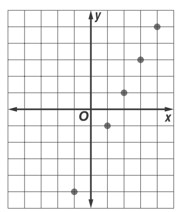
Solve each equation for the given domain. Graph the solution set.

13. $y = 2x + 5$ for x = {−5, −4, −2, −1, 0}



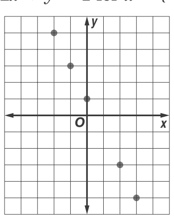
{(−5, −5), (−4, −3), (−2, 1), (−1, 3), (0, 5)}

14. $y = 2x - 3$ for x = {−1, 1, 2, 3, 4}



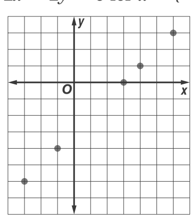
{(−1, −5), (1, −1), (2, 1), (3, 3), (4, 5)}

15. $2x + y = 1$ for x = {−2, −1, 0, 2, 3}



{(−2, 5), (−1, 3), (0, 1), (2, −3), (3, −5)}

16. $2x - 2y = 6$ for x = {−3, −1, 3, 4, 6}



{(−3, −6), (−1, −4), (3, 0), (4, 1), (6, 3)}

Lesson 4-4

NAME ______________________ DATE ________ PERIOD _____

4-4 Practice (Average)

Equations as Relations

Find the solution set for each equation, given the replacement set.

1. $y = 2 - 5x$; {(3, 12), (−3, −17), (2, −8), (−1, 7)} **{(2, −8), (−1, 7)}**

2. $3x - 2y = -1$; {(−1, 1), (−2, −2.5), (−1, −1.5), (0, 0.5)} **{(−2, −2.5), (0, 0.5)}**

Solve each equation if the domain is {−2, −1, 2, 3, 5}.

3. $y = 4 - 2x$ **{(−2, 8), (−1, 6), (2, 0), (3, −2), (5, −6)}**

4. $x = 8 - y$ **{(−2, 10), (−1, 9), (2, 6), (3, 5), (5, 3)}**

5. $4x + 2y = 10$ **{(−2, 9), (−1, 7), (2, 1), (3, −1), (5, −5)}**

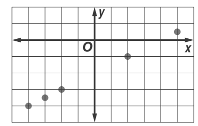
6. $3x - 6y = 12$ **{(−2, −3), (−1, −2.5), (2, −1), (3, −0.5), (5, 0.5)}**

7. $2x + 4y = 16$ **{(−2, 5), (−1, 4.5), (2, 3), (3, 2.5), (5, 1.5)}**

8. $x - \frac{1}{2}y = 6$ **{(−2, −16), (−1, −14), (2, −8), (3, −6), (5, −2)}**

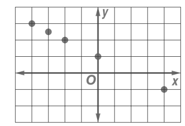
Solve each equation for the given domain. Graph the solution set.

9. $2x - 4y = 8$ for x = {−4, −3, −2, 2, 5}



{(−4, −4), (−3, −3.5), (−2, −3), (2, −1), (5, 0.5)}

10. $\frac{1}{2}x + y = 1$ for x = {−4, −3, −2, 0, 4}



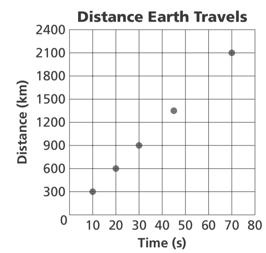
{(−4, 3), $\left(-3, 2\frac{1}{2}\right)$, (−2, 2), (0, 1), (4, −1)}

EARTH SCIENCE For Exercises 11 and 12, use the following information.

Earth moves at a rate of 30 kilometers per second around the Sun. The equation $d = 30t$ relates the distance d in kilometers Earth moves to time t in seconds.

11. Find the set of ordered pairs when t = {10, 20, 30, 45, 70}. **{(10, 300), (20, 600), (30, 900), (45, 1350), (70, 2100)}**

12. Graph the set of ordered pairs.



GEOMETRY For Exercises 13–15, use the following information.

The equation for the area of a triangle is $A = \frac{1}{2}bh$. Suppose the area of triangle DEF is 30 square inches.

13. Solve the equation for h. $h = \frac{60}{b}$

14. State the independent and dependent variables. ***b* is independent; *h* is dependent.**

15. Choose 5 values for b and find the corresponding values for h.
Sample answer: {(5, 12), (6, 10), (10, 6), (12, 5), (15, 4)}

NAME ______________________ DATE __________ PERIOD _____

4-4 Reading to Learn Mathematics

Equations as Relations

Pre-Activity **Why are equations of relations important in traveling?**

Read the introduction to Lesson 4-4 at the top of page 212 in your textbook.

- In the equation $p = 0.69d$, p represents **pounds** and d represents **dollars**.
- How many variables are in the equation $p = 0.69d$? **two**

Reading the Lesson

1. Suppose you make the following table to solve an equation that uses the domain $\{-3, -2, -1, 0, 1\}$.

x	$x - 4$	y	(x, y)
−3	−3 − 4	−7	(−3, −7)
−2	−2 − 4	−6	(−2, −6)
−1	−1 − 4	−5	(−1, −5)
0	0 − 4	−4	(0, −4)
1	1 − 4	−3	(1, −3)

a. What is the equation? **$y = x - 4$**

b. Which column shows the *domain*? **the first column**

c. Which column shows the *range*? **the third column**

d. Which column shows the *solution set*? **the fourth column**

2. The solution set of the equation $y = 2x$ for a given domain is $\{(-2, -4), (0, 0), (2, 4), (7, 14)\}$. Tell whether each sentence is *true* or *false*. If false, replace the underlined word(s) to make a true sentence.

a. The domain contains the values represented by the <u>independent variable</u>. **true**

b. The <u>domain</u> contains the numbers −4, 0, 4, and 14. **false; range**

c. For each number in the domain, the range contains a corresponding number that is a value of the <u>dependent variable</u>. **true**

3. What is meant by "solving an equation for y in terms of x"? **isolating y on one side of the equation**

Helping You Remember

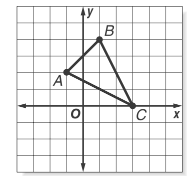
4. Remember, *when you solve an equation for a given variable, that variable becomes the dependent variable*. Write an equation and describe how you would identify the dependent variable. **Sample answer: $y = 3x$ shows that you have solved for y, so y is the dependent variable.**

NAME ______________________ DATE __________ PERIOD _____

4-4 Enrichment

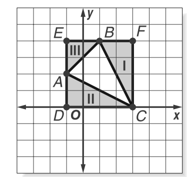
Coordinate Geometry and Area

How would you find the area of a triangle whose vertices have the coordinates $A(-1, 2)$, $B(1, 4)$, and $C(3, 0)$?



When a figure has no sides parallel to either axis, the height and base are difficult to find.

One method of finding the area is to enclose the figure in a rectangle and subtract the area of the surrounding triangles from the area of the rectangle.



Area of rectangle $DEFC$
$= 4 \times 4$
$= 16$ square units

Area of triangle I $= \frac{1}{2}(2)(4) = 4$

Area of triangle II $= \frac{1}{2}(2)(4) = 4$

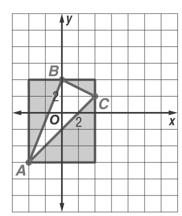
Area of triangle III $= \frac{1}{2}(2)(2) = 2$

Total = 10 square units

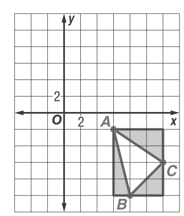
Area of triangle $ABC = 16 - 10$, or 6 square units

Find the areas of the figures with the following vertices.

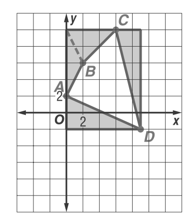
1. $A(-4, -6)$, $B(0, 4)$, $C(4, 2)$
24 square units



2. $A(6, -2)$, $B(8, -10)$, $C(12, -6)$
20 square units



3. $A(0, 2)$, $B(2, 7)$, $C(6, 10)$, $D(9, -2)$
55 square units



Lesson 4-4

Answers

NAME ______________________ DATE ________ PERIOD _____

4-5 Study Guide and Intervention

Graphing Linear Equations

Identify Linear Equations A **linear equation** is an equation that can be written in the form $Ax + By = C$. This is called the **standard form** of a linear equation.

Standard Form of a Linear Equation	$Ax + By = C$, where $A \geq 0$, A and B are not both zero, and A, B, and C are integers whose GCF is 1.

Example 1 **Determine whether $y = 6 - 3x$ is a linear equation. If so, write the equation in standard form.**

First rewrite the equation so both variables are on the same side of the equation.

$y = 6 - 3x$ Original equation

$y + 3x = 6 - 3x + 3x$ Add 3x to each side.

$3x + y = 6$ Simplify.

The equation is now in standard form, with $A = 3$, $B = 1$ and $C = 6$. This is a linear equation.

Example 2 **Determine whether $3xy + y = 4 + 2x$ is a linear equation. If so, write the equation in standard form.**

Since the term $3xy$ has two variables, the equation cannot be written in the form $Ax + By = C$. Therefore, this is not a linear equation.

Exercises

Determine whether each equation is a linear equation. If so, write the equation in standard form.

1. $2x = 4y$
yes; $2x - 4y = 0$

2. $6 + y = 8$
yes; $y = 2$

3. $4x - 2y = -1$
yes; $4x - 2y = -1$

4. $3xy + 8 = 4y$
no

5. $3x - 4 = 12$
yes; $3x = 16$

6. $y = x^2 + 7$
no

7. $y - 4x = 9$
yes; $4x - y = -9$

8. $x + 8 = 0$
yes; $x = -8$

9. $-2x + 3 = 4y$
yes; $2x + 4y = 3$

10. $2 + \frac{1}{2}x = y$
yes; $x - 2y = -4$

11. $\frac{1}{4}y = 12 - 4x$
yes; $16x + y = 48$

12. $3xy - y = 8$
no

13. $6x + 4y - 3 = 0$
yes; $6x + 4y = 3$

14. $yx - 2 = 8$
no

15. $6a - 2b = 8 + b$
yes; $6a - 3b = 8$

16. $\frac{1}{4}x - 12y = 1$
yes; $x - 48y = 4$

17. $3 + x + x^2 = 0$
no

18. $x^2 = 2xy$
no

Lesson 4-5

NAME ______________________ DATE ________ PERIOD _____

4-5 Study Guide and Intervention *(continued)*

Graphing Linear Equations

Graph Linear Equations The graph of a linear equation is a line. The line represents all solutions to the linear equation. Also, every ordered pair on this line satisfies the equation.

Example **Graph the equation $y - 2x = 1$.**

Solve the equation for y.

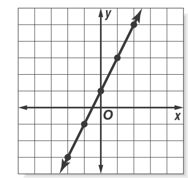
$y - 2x = 1$ Original equation

$y - 2x + 2x = 1 + 2x$ Add 2x to each side.

$y = 2x + 1$ Simplify.

Select five values for the domain and make a table. Then graph the ordered pairs and draw a line through the points.

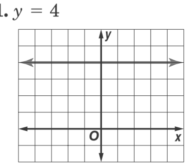
x	2x + 1	y	(x, y)
−2	2(−2) + 1	−3	(−2, −3)
−1	2(−1) + 1	−1	(−1, −1)
0	2(0) + 1	1	(0, 1)
1	2(1) + 1	3	(1, 3)
2	2(2) + 1	5	(2, 5)



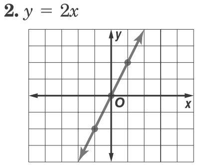
Exercises

Graph each equation.

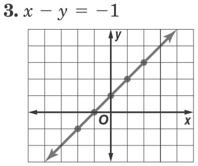
1. $y = 4$



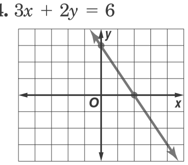
2. $y = 2x$



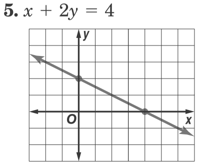
3. $x - y = -1$



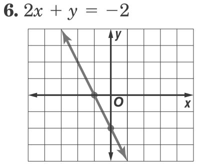
4. $3x + 2y = 6$



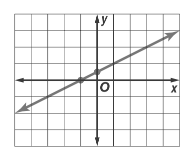
5. $x + 2y = 4$



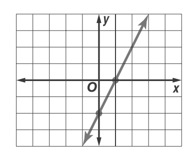
6. $2x + y = -2$



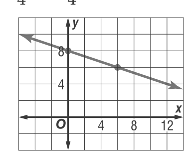
7. $3x - 6y = -3$



8. $-2x + y = -2$



9. $\frac{1}{4}x + \frac{3}{4}y = 6$



NAME ______________________ DATE ____________ PERIOD _____

4-5 Skills Practice

Graphing Linear Equations

Determine whether each equation is a linear equation. If so, write the equation in standard form.

1. $xy = 6$
no

2. $y = 2 - 3x$
yes; $3x + y = 2$

3. $5x = y - 4$
yes; $5x - y = -4$

4. $y = 2x + 5$
yes; $2x - y = -5$

5. $y = -7 + 6x$
yes; $6x - y = 7$

6. $y = 3x^2 + 1$
no

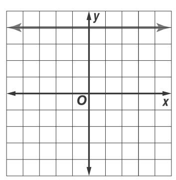
7. $y - 4 = 0$
yes; $y = 4$

8. $5x + 6y = 3x + 2$
yes; $x + 3y = 1$

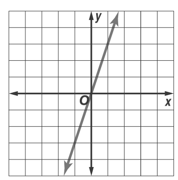
9. $\frac{1}{2}y = 1$
yes; $y = 2$

Graph each equation.

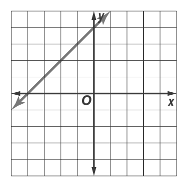
10. $y = 4$



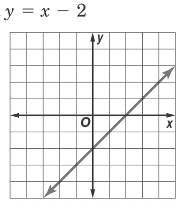
11. $y = 3x$



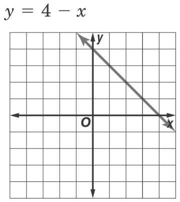
12. $y = x + 4$



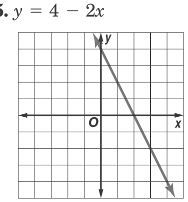
13. $y = x - 2$



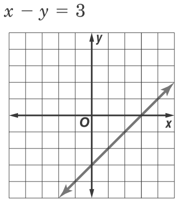
14. $y = 4 - x$



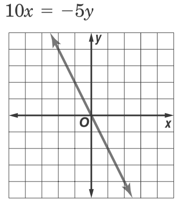
15. $y = 4 - 2x$



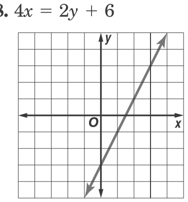
16. $x - y = 3$



17. $10x = -5y$



18. $4x = 2y + 6$



NAME ______________________ DATE ____________ PERIOD _____

4-5 Practice (Average)

Graphing Linear Equations

Determine whether each equation is a linear equation. If so, write the equation in standard form.

1. $4xy + 2y = 9$
no

2. $8x - 3y = 6 - 4x$
yes; $4x - y = 2$

3. $7x + y + 3 = y$
yes; $7x = -3$

4. $5 - 2y = 3x$
yes; $3x + 2y = 5$

5. $4y + x = 9x$
yes; $8x - 4y = 0$

6. $a + \frac{1}{5}b = 2$
yes; $5a + b = 10$

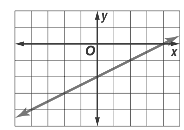
7. $6x = 2y$
yes; $6x - 2y = 0$

8. $\frac{x}{4} - \frac{y}{3} = 1$
yes; $3x - 4y = 12$

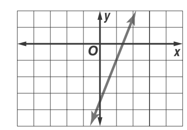
9. $\frac{5}{x} - \frac{2}{y} = 7$
no

Graph each equation.

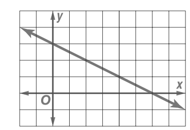
10. $\frac{1}{2}x - y = 2$



11. $5x - 2y = 7$



12. $1.5x + 3y = 9$



COMMUNICATIONS For Exercises 13–15, use the following information.

A telephone company charges \$4.95 per month for long distance calls plus \$0.05 per minute. The monthly cost c of long distance calls can be described by the equation $c = 0.05m + 4.95$, where m is the number of minutes.

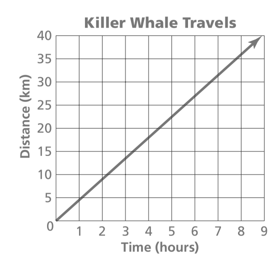
13. Find the y-intercept of the graph of the equation.
(0, 4.95)

14. Graph the equation.

15. If you talk 140 minutes, what is the monthly cost for long distance? **\$11.95**

MARINE BIOLOGY For Exercises 16 and 17, use the following information.

Killer whales usually swim at a rate of 3.2–9.7 kilometers per hour, though they can travel up to 48.4 kilometers per hour. Suppose a migrating killer whale is swimming at an average rate of 4.5 kilometers per hour. The distance d the whale has traveled in t hours can be predicted by the equation $d = 4.5t$.



16. Graph the equation.

17. Use the graph to predict the time it takes the killer whale to travel 30 kilometers. **between 6 h and 7 h**

NAME ______________________ DATE ________ PERIOD _____

4-5 Reading to Learn Mathematics

Graphing Linear Equations

Pre-Activity **How can linear equations be used in nutrition?**

Read the introduction to Lesson 4-5 at the top of page 218 in your textbook.

In the equation $f = 0.3\left(\frac{C}{9}\right)$, what are the independent and dependent variables? **C is independent, f is dependent.**

Reading the Lesson

1. Describe the graph of a linear equation. **The graph is a straight line.**

2. Determine whether each equation is a linear equation. Explain.

	Equation	Linear or non-linear?	Explanation
a.	$2x = 3y + 1$	linear	The equation can be written as $2x - 3y = 1$.
b.	$4xy + 2y = 7$	non-linear	$4xy$ has two variables.
c.	$2x^2 = 4y - 3$	non-linear	The variable x has an exponent of 2.
d.	$\frac{x}{5} - \frac{4y}{3} = 2$	linear	The equation can be written as $3x - 20y = 30$.

3. What do the terms *x-intercept* and *y-intercept* mean? **The x-intercept is the x-coordinate of the point where the graph of an equation crosses the x-axis, and the y-intercept is the y-coordinate of the point where the graph crosses the y-axis.**

Helping You Remember

4. Describe the method you would use to graph $4x + 2y = 8$. **Sample answer: Find the x- and y-intercepts, which are 2 and 4. Plot the points for the ordered pairs (2, 0) and (0, 4), and draw a line that connects the points.**

Lesson 4-5

NAME ______________________ DATE ________ PERIOD _____

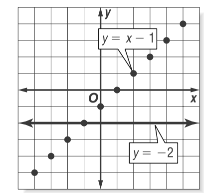
4-5 Enrichment

Taxicab Graphs

You have used a rectangular coordinate system to graph equations such as $y = x - 1$ on a coordinate plane. In a coordinate plane, the numbers in an ordered pair (x, y) can be any two real numbers.

A **taxicab plane** is different from the usual coordinate plane. The only points allowed are those that exist along the horizontal and vertical grid lines. You may think of the points as taxicabs that must stay on the streets.

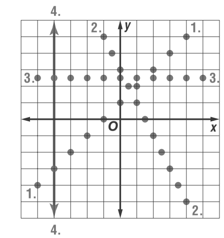
The taxicab graph shows the equations $y = -2$ and $y = x - 1$. Notice that one of the graphs is no longer a straight line. It is now a collection of separate points.



Graph these equations on the taxicab plane at the right.

1. $y = x + 1$ **2.** $y = -2x + 3$

3. $y = 2.5$ **4.** $x = -4$



Use your graphs for these problems.

5. Which of the equations has the same graph in both the usual coordinate plane and the taxicab plane? **$x = -4$**

6. Describe the form of equations that have the same graph in both the usual coordinate plane and the taxicab plane.
$x = A$ and $y = B$, where A and B are integers

In the taxicab plane, distances are not measured diagonally, but along the streets. Write the taxi-distance between each pair of points.

7. (0, 0) and (5, 2) **7 units**

8. (0, 0) and (-3, 2) **5 units**

9. (0, 0) and (2, 1.5) **3.5 units**

10. (1, 2) and (4, 3) **4 units**

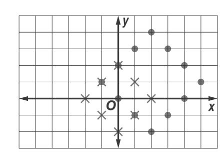
11. (2, 4) and (-1, 3) **4 units**

12. (0, 4) and (-2, 0) **6 units**

Draw these graphs on the taxicab grid at the right.

13. The set of points whose taxi-distance from (0, 0) is 2 units. **indicated by crosses**

14. The set of points whose taxi-distance from (2, 1) is 3 units. **indicated by dots**



NAME ________________ DATE ________ PERIOD ____

4-6 Study Guide and Intervention
Functions

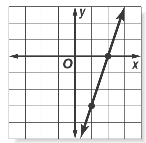
Identify Functions Relations in which each element of the domain is paired with exactly one element of the range are called **functions**.

Example 1 **Determine whether the relation {(6, −3), (4, 1), (7, −2), (−3, 1)} is a function. Explain.**

Since each element of the domain is paired with exactly one element of the range, this relation is a function.

Example 2 **Determine whether $3x - y = 6$ is a function.**

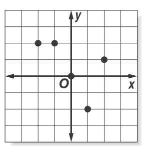
Since the equation is in the form $Ax + By = C$, the graph of the equation will be a line, as shown at the right.



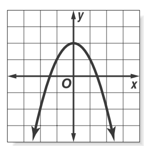
If you draw a vertical line through each value of x, the vertical line passes through just one point of the graph. Thus, the line represents a function.

Exercises

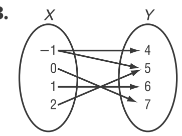
Determine whether each relation is a function.

1. 

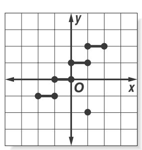
yes

2. 

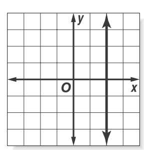
yes

3. 

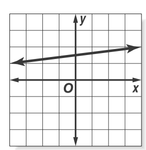
no

4. 

no

5. 

no

6. 

yes

7. {(4, 2), (2, 3), (6, 1)}
yes

8. {(−3, −3), (−3, 4), (−2, 4)}
no

9. {(−1, 0), (1, 0)}
yes

10. $-2x + 4y = 0$
yes

11. $x^2 + y^2 = 8$
no

12. $x = -4$
no

NAME ________________ DATE ________ PERIOD ____

4-6 Study Guide and Intervention *(continued)*
Functions

Function Values Equations that are functions can be written in a form called **function notation**. For example, $y = 2x - 1$ can be written as $f(x) = 2x - 1$. In the function, x represents the elements of the domain, and $f(x)$ represents the elements of the range. Suppose you want to find the value in the range that corresponds to the element 2 in the domain. This is written $f(2)$ and is read "f of 2." The value of $f(2)$ is found by substituting 2 for x in the equation.

Example **If $f(x) = 3x - 4$, find each value.**

a. $f(3)$

$f(3) = 3(3) - 4$ Replace x with 3.
$= 9 - 4$ Multiply.
$= 5$ Simplify.

b. $f(-2)$

$f(-2) = 3(-2) - 4$ Replace x with −2.
$= -6 - 4$ Multiply.
$= -10$ Simplify.

Exercises

If $f(x) = 2x - 4$ and $g(x) = x^2 - 4x$, find each value.

1. $f(4)$
4

2. $g(2)$
−4

3. $f(-5)$
−14

4. $g(-3)$
21

5. $f(0)$
−4

6. $g(0)$
0

7. $f(3) - 1$
1

8. $f\left(\frac{1}{4}\right)$
$-3\frac{1}{2}$

9. $g\left(\frac{1}{4}\right)$
$-\frac{15}{16}$

10. $f(a^2)$
$2a^2 - 4$

11. $f(k + 1)$
$2k - 2$

12. $g(2c)$
$4c^2 - 8c$

13. $f(3x)$
$6x - 4$

14. $f(2) + 3$
3

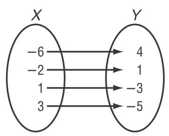
15. $g(-4)$
32

NAME ______________________ DATE ____________ PERIOD _____

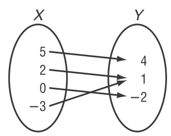
4-6 Skills Practice

Functions

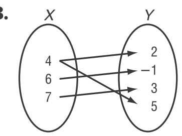
Determine whether each relation is a function.

1. 

yes

2. 

yes

3. 

no

4. yes

x	y
4	−5
−1	−10
0	−9
1	−7
9	1

5. no

x	y
2	7
5	−3
3	5
−4	−2
5	2

6. no

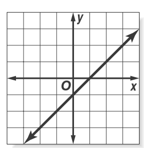
x	y
3	7
−1	1
1	0
3	5
7	3

7. $\{(2, 5), (4, -2), (3, 3), (5, 4), (-2, 5)\}$ **yes**

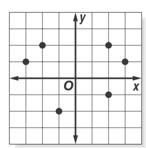
8. $\{(6, -1), (-4, 2), (5, 2), (4, 6), (6, 5)\}$ **no**

9. $y = 2x - 5$ **yes**

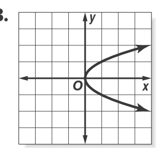
10. $y = 11$ **yes**

11. 

yes

12. 

no

13. 

no

If $f(x) = 3x + 2$ and $g(x) = x^2 - x$, find each value.

14. $f(4)$ **14**

15. $f(8)$ **26**

16. $f(-2)$ **−4**

17. $g(2)$ **2**

18. $g(-3)$ **12**

19. $g(-6)$ **42**

20. $f(2) + 1$ **9**

21. $f(1) - 1$ **4**

22. $g(2) - 2$ **0**

23. $g(-1) + 4$ **6**

24. $f(x + 1)$ **$3x + 5$**

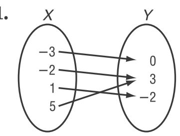
25. $g(3b)$ **$9b^2 - 3b$**

NAME ______________________ DATE ____________ PERIOD _____

4-6 Practice (Average)

Functions

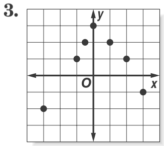
Determine whether each relation is a function.

1. 

yes

2. no

x	y
1	−5
−4	3
7	6
1	−2

3. 

yes

4. $\{(1, 4), (2, -2), (3, -6), (-6, 3), (-3, 6)\}$ **yes**

5. $\{(6, -4), (2, -4), (-4, 2), (4, 6), (2, 6)\}$ **no**

6. $x = -2$ **no**

7. $y = 2$ **yes**

If $f(x) = 2x - 6$ and $g(x) = x - 2x^2$, find each value.

8. $f(2)$ **−2**

9. $f\left(-\frac{1}{2}\right)$ **−7**

10. $g(-1)$ **−3**

11. $g\left(-\frac{1}{3}\right)$ **$-\frac{5}{9}$**

12. $f(7) - 9$ **−1**

13. $g(-3) + 13$ **−8**

14. $f(h + 9)$ **$2h + 12$**

15. $g(3y)$ **$3y - 18y^2$**

16. $2[g(b) + 1]$ **$2b - 4b^2 + 2$**

WAGES For Exercises 17 and 18, use the following information.

Martin earns $7.50 per hour proofreading ads at a local newspaper. His weekly wage w can be described by the equation $w = 7.5h$, where h is the number of hours worked.

17. Write the equation in functional notation. **$f(h) = 7.5h$**

18. Find $f(15)$, $f(20)$, and $f(25)$. **112.50, 150, 187.50**

ELECTRICITY For Exercises 19–21, use the following information.

The table shows the relationship between resistance R and current I in a circuit.

Resistance (ohms)	120	80	48	6	4
Current (amperes)	0.1	0.15	0.25	2	3

19. Is the relationship a function? Explain. **Yes; for each value in the domain, there is only one value in the range.**

20. If the relation can be represented by the equation $IR = 12$, rewrite the equation in functional notation so that the resistance R is a function of the current I. **$f(I) = \frac{12}{I}$**

21. What is the resistance in a circuit when the current is 0.5 ampere? **24 ohms**

NAME ______________________ DATE __________ PERIOD _____

4-6 Reading to Learn Mathematics

Functions

Pre-Activity **How are functions used in meteorology?**

Read the introduction to Lesson 4-6 at the top of page 226 in your textbook.

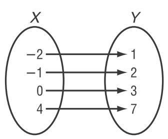
If pressure is the independent variable and temperature is the dependent variable, what are the ordered pairs for this set of data?
{(1013, 3), (1006, 4), (997, 10), (995, 13), (995, 8), (1000, 4), (1006, 1), (1011, −2), (1016, −6), (1019, −9)}

Reading the Lesson

1. The statement, "Relations in which each element of the range is paired with exactly one element of the domain are called functions," is false. How can you change the underlined words to make the statement true? **Change *range* to *domain* and *domain* to *range*.**

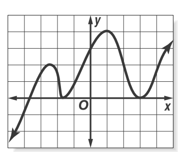
2. Describe how each method shows that the relation represented is a function.

a. mapping



The elements of the domain are paired with corresponding elements in the range. Each element of the domain has only one arrow going from it.

b. vertical line test



No vertical line passes through more than one point of the graph.

Helping You Remember

3. A student who was trying to help a friend remember how functions are different from relations that are not functions gave the following advice: *Just remember that functions are very strict and never give you a choice*. Explain how this might help you remember what a function is. **Sample answer: A function always pairs each element in the domain with exactly one element in the range. If two people start with the same element of the domain, they are forced to pair it with the same element in the range. The second person cannot pick a different number from the first person.**

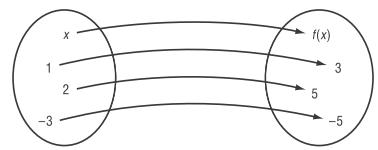
NAME ______________________ DATE __________ PERIOD _____

4-6 Enrichment

Composite Functions

Three things are needed to have a function—a set called the domain, a set called the range, and a rule that matches each element in the domain with only one element in the range. Here is an example.

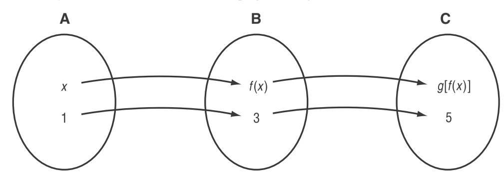
Rule: $f(x) = 2x + 1$



$f(x) = 2x + 1$
$f(1) = 2(1) + 1 = 2 + 1 = 3$
$f(2) = 2(2) + 1 = 4 + 1 = 5$
$f(-3) = 2(-3) + 1 = -6 + 1 = -5$

Suppose we have three sets A, B, and C and two functions described as shown below.

Rule: $f(x) = 2x + 1$ Rule: $g(y) = 3y - 4$



$g(y) = 3y - 4$
$g(3) = 3(3) - 4 = 5$

Let's find a rule that will match elements of set A with elements of set C without finding any elements in set B. In other words, let's find a rule for the **composite function $g[f(x)]$**.

Since $f(x) = 2x + 1$, $g[f(x)] = g(2x + 1)$.
Since $g(y) = 3y - 4$, $g(2x + 1) = 3(2x + 1) - 4$, or $6x - 1$.
Therefore, $g[f(x)] = 6x - 1$.

Find a rule for the composite function $g[f(x)]$.

1. $f(x) = 3x$ and $g(y) = 2y + 1$
$g[f(x)] = 6x + 1$

2. $f(x) = x^2 + 1$ and $g(y) = 4y$
$g[f(x)] = 4x^2 + 4$

3. $f(x) = -2x$ and $g(y) = y^2 - 3y$
$g[f(x)] = 4x^2 + 6x$

4. $f(x) = \frac{1}{x - 3}$ and $g(y) = y^{-1}$
$g[f(x)] = x - 3$

5. Is it always the case that $g[f(x)] = f[g(x)]$? Justify your answer.
No. For example, in Exercise 1, $f[g(x)] = f(2x + 1) = 3(2x + 1) = 6x + 3$, not $6x + 1$.

NAME ______________________ DATE ________ PERIOD _____

4-7 Study Guide and Intervention

Arithmetic Sequences

Recognize Arithmetic Sequences A **sequence** is a set of numbers in a specific order. If the difference between successive terms is constant, then the sequence is called an **arithmetic sequence**.

Arithmetic Sequence	a numerical pattern that increases or decreases at a constant rate or value called the **common difference**

Example 1 **Determine whether the sequence 1, 3, 5, 7, 9, 11, … is an arithmetic sequence. Justify your answer.**

If possible, find the common difference between the terms. Since $3 - 1 = 2$, $5 - 3 = 2$, and so on, the common difference is 2.

Since the difference between the terms of 1, 3, 5, 7, 9, 11, … is constant, this is an arithmetic sequence.

Example 2 **Determine whether the sequence 1, 2, 4, 8, 16, 32, … is an arithmetic sequence. Justify your answer.**

If possible, find the common difference between the terms. Since $2 - 1 = 1$ and $4 - 2 = 2$, there is no common difference.

Since the difference between the terms of 1, 2, 4, 8, 16, 32, … is not constant, this is not an arithmetic sequence.

Exercises

Determine whether each sequence is an arithmetic sequence. If it is, state the common difference.

1. 1, 5, 9, 13, 17, … **yes; 4**

2. 8, 4, 0, −4, −8, … **yes; −4**

3. 1, 3, 9, 27, 81, … **no**

4. 10, 15, 25, 40, 60, … **no**

5. −10, −5, 0, 5, 10, … **yes; 5**

6. 8, 6, 4, 2, 0, −2, … **yes; −2**

7. 4, 8, 12, 16, … **yes; 4**

8. 15, 12, 10, 9, … **no**

9. 1.1, 2.1, 3.1, 4.1, 5.1, … **yes; 1**

10. 8, 7, 6, 5, 4, … **yes; −1**

11. 0.5, 1.5, 2.5, 3.5, 4.5, … **yes; 1**

12. 1, 4, 16, 64, … **no**

13. 10, 14, 18, 22, … **yes; 4**

14. −3, −6, −9, −12, … **yes; −3**

15. 7, 0, −7, −14, … **yes; −7**

Lesson 4-7

NAME ______________________ DATE ________ PERIOD _____

4-7 Study Guide and Intervention *(continued)*

Arithmetic Sequences

Write Arithmetic Sequences You can use the common difference of an arithmetic sequence to find the next term of the sequence. Each term after the first term is found by adding the preceding term and the common difference.

Terms of an Arithmetic Sequence	If a_1 is the first term of an arithmetic sequence with common difference d, then the sequence is $a_1, a_1 + d, a_1 + 2d, a_1 + 3d, \ldots$.
***n*th Term of an Arithmetic Sequence**	$a_n = a_1 + (n - 1)d$

Example 1 **Find the next three terms of the arithmetic sequence 28, 32, 36, 40, … .**

Find the common difference by subtracting successive terms.

$$28 \underset{+4}{\frown} 32 \underset{+4}{\frown} 36 \underset{+4}{\frown} 40$$

The common difference is 4.

Add 4 to the last given term, 40, to get the next term. Continue adding 4 until the next three terms are found.

$$40 \underset{+4}{\frown} 44 \underset{+4}{\frown} 48 \underset{+4}{\frown} 52$$

The next three terms are 44, 48, 52.

Example 2 **Write an equation for the *n*th term of the sequence 12, 15, 18, 21, … .**

In this sequence, a_1 is 12. Find the common difference.

$$12 \underset{+3}{\frown} 15 \underset{+3}{\frown} 18 \underset{+3}{\frown} 21$$

The common difference is 3.

Use the formula for the nth term to write an equation.

$a_n = a_1 + (n - 1)d$ Formula for the *n*th term

$a_n = 12 + (n - 1)3$ $a_1 = 12, d = 3$

$a_n = 12 + 3n - 3$ Distributive Property

$a_n = 3n + 9$ Simplify.

The equation for the nth term is $a_n = 3n + 9$.

Exercises

Find the next three terms of each arithmetic sequence.

1. 9, 13, 17, 21, 25, … **29, 33, 37**

2. 4, 0, −4, −8, −12, … **−16, −20, −24**

3. 29, 35, 41, 47, … **53, 59, 65**

4. −10, −5, 0, 5, … **10, 15, 20**

5. 2.5, 5, 7.5, 10, … **12.5, 15, 17.5**

6. 3.1, 4.1, 5.1, 6.1, … **7.1, 8.1, 9.1**

Find the *n*th term of each arithmetic sequence described.

7. $a_1 = 6, d = 3, n = 10$ **33**

8. $a_1 = -2, d = -3, n = 8$ **−23**

9. $a_1 = 1, d = -5, n = 20$ **−94**

10. $a_1 = -3, d = -2, n = 50$ **−101**

11. $a_1 = -12, d = 4, n = 20$ **64**

12. $a_1 = 1, d = \frac{1}{2}, n = 11$ **6**

Write an equation for the *n*th term of the arithmetic sequence.

13. 1, 3, 5, 7, … $a_n = 2n - 1$

14. −1, −4, −7, −10, … $a_n = -3n + 2$

15. −4, −9, −14, −19, … $a_n = -5n + 1$

NAME ______________________ DATE ____________ PERIOD _____

4-7 Skills Practice

Arithmetic Sequences

Determine whether each sequence is an arithmetic sequence. If it is, state the common difference.

1. 4, 7, 9, 12, … no

2. 15, 13, 11, 9, … yes; −2

3. 7, 10, 13, 16, … yes; 3

4. −6, −5, −3, −1, … no

5. −5, −3, −1, 1, … yes; 2

6. −9, −12, −15, −18, … yes; −3

Find the next three terms of each arithmetic sequence.

7. 3, 7, 11, 15, … 19, 23, 27

8. 22, 20, 18, 16, … 14, 12, 10

9. −13, −11, −9, −7, … −5, −3, −1

10. −2, −5, −8, −11, … −14, −17, −20

11. 19, 24, 29, 34, … 39, 44, 49

12. 16, 7, −2, −11, … −20, −29, −38

Find the *n*th term of each arithmetic sequence described.

13. $a_1 = 6, d = 3, n = 12$ 39

14. $a_1 = -2, d = 5, n = 11$ 48

15. $a_1 = 10, d = -3, n = 15$ −32

16. $a_1 = -3, d = -3, n = 22$ −66

17. $a_1 = 24, d = 8, n = 25$ 216

18. $a_1 = 8, d = -6, n = 14$ −70

19. 8, 13, 18, 23, … for $n = 17$ 88

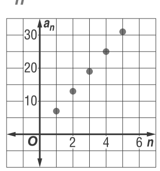
20. −10, −3, 4, 11, … for $n = 12$ 67

21. 12, 10, 8, 6, … for $n = 16$ −18

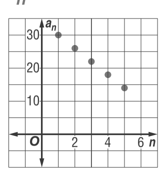
22. 12, 7, 2, −3, … for $n = 25$ −108

Write an equation for the *n*th term of each arithmetic sequence. Then graph the first five terms of the sequence.

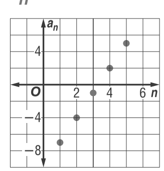
23. 7, 13, 19, 25, …
$a_n = 6n + 1$



24. 30, 26, 22, 18, …
$a_n = -4n + 34$



25. −7, −4, −1, 2, …
$a_n = 3n - 10$



Lesson 4-7

NAME ______________________ DATE ____________ PERIOD _____

4-7 Practice (Average)

Arithmetic Sequences

Determine whether each sequence is an arithmetic sequence. If it is, state the common difference.

1. 21, 13, 5, −3, … yes; −8

2. −5, 12, 29, 46, … yes; 17

3. −2.2, −1.1, 0.1, 1.3, … no

Find the next three terms of each arithmetic sequence.

4. 82, 76, 70, 64, … 58, 52, 46

5. −49, −35, −21, −7, … 7, 21, 35

6. $\frac{3}{4}, \frac{1}{2}, \frac{1}{4}, 0, \ldots$ $-\frac{1}{4}, -\frac{1}{2}, -\frac{3}{4}$

Find the *n*th term of each arithmetic sequence described.

7. $a_1 = 7, d = 9, n = 18$ 160

8. $a_1 = -12, d = 4, n = 36$ 128

9. −18, −13, −8, −3, … for $n = 27$ 112

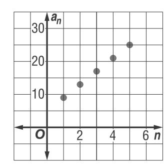
10. 4.1, 4.8, 5.5, 6.2, … for $n = 14$ 13.2

11. $a_1 = \frac{3}{8}, d = \frac{1}{4}, n = 15$ $3\frac{7}{8}$

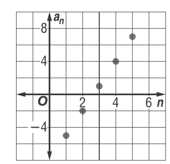
12. $a_1 = 2\frac{1}{2}, d = 1\frac{1}{2}, n = 24$ 37

Write an equation for the *n*th term of each arithmetic sequence. Then graph the first five terms of the sequence.

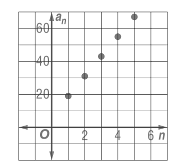
13. 9, 13, 17, 21, …
$a_n = 4n + 5$



14. −5, −2, 1, 4, …
$a_n = 3n - 8$



15. 19, 31, 43, 55, …
$a_n = 12n + 7$



BANKING For Exercises 16 and 17, use the following information.

Chem deposited $115.00 in a savings account. Each week thereafter, he deposits $35.00 into the account.

16. Write a formula to find the total amount Chem has deposited for any particular number of weeks after his initial deposit. $a_n = 35n + 115$

17. How much has Chem deposited 30 weeks after his initial deposit? $1165

18. STORE DISPLAY Tamika is stacking boxes of tissue for a store display. Each row of tissues has 2 fewer boxes than the row below. The first row has 23 boxes of tissues. How many boxes will there be in the tenth row? 5

Answers

NAME ______________________ DATE __________ PERIOD _____

4-7 Reading to Learn Mathematics

Arithmetic Sequences

Pre-Activity **How are arithmetic sequences used to solve problems in science?**

Read the introduction to Lesson 4-7 at the top of page 233 in your textbook.

Describe the pattern in the data. **The altitude of the probe increases by 8.2 feet each second.**

Reading the Lesson

1. Do the recorded altitudes in the introduction form an arithmetic sequence? Explain. **Yes; the difference between the successive terms is the constant 8.2.**

2. What is meant by *successive terms*? **terms that come one right after the other**

3. Complete the table.

	Pattern	Is the sequence increasing or decreasing?	Is there a common difference? If so, what is it?
a.	2, 5, 8, 11, 14, …	increasing	yes; 3
b.	55, 50, 45, 40, …	decreasing	yes; −5
c.	1, 2, 4, 9, 16, …	increasing	no
d.	$\frac{1}{2}, 0, -\frac{1}{2}, -1, \ldots$	decreasing	yes; $-\frac{1}{2}$
e.	2.6, 2.9, 3.2, 3.5, …	increasing	yes; 0.3

Helping You Remember

4. Use the pattern 3, 7, 11, 15, … to explain how you would help someone else learn how to find the 10th term of an arithmetic sequence. **Find the common difference of 4. To find the 10th term, use the formula $a_n = a_1 + (n - 1)d$. Substitute 10 for n, 3 for a_1, and 4 for d. The 10th term is 39.**

Lesson 4-7

NAME ______________________ DATE __________ PERIOD _____

4-7 Enrichment

Arithmetic Series

An arithmetic series is a series in which each term after the first may be found by adding the same number to the preceding term. Let S stand for the following series in which each term is 3 more than the preceding one.

$S = 2 + 5 + 8 + 11 + 14 + 17 + 20$

The series remains the same if we reverse the order of all the terms. So let us reverse the order of the terms and add one series to the other, term by term. This is shown at the right.

$$\begin{array}{rl} S = & 2 + 5 + 8 + 11 + 14 + 17 + 20 \\ S = & 20 + 17 + 14 + 11 + 8 + 5 + 2 \\ \hline 2S = & 22 + 22 + 22 + 22 + 22 + 22 + 22 \\ 2S = & 7(22) \\ S = & \frac{7(22)}{2} = 7(11) = 77 \end{array}$$

Let a represent the first term of the series.

Let ℓ represent the last term of the series.

Let n represent the number of terms in the series.

In the preceding example, a = 2, l = 20, and $n = 7$. Notice that when you add the two series, term by term, the sum of each pair of terms is 22. That sum can be found by adding the first and last terms, 2 + 20 or $a + \ell$. Notice also that there are 7, or n, such sums. Therefore, the value of $2S$ is 7(22), or $n(a + \ell)$ in the general case. Since this is twice the sum of the series, you can use the following formula to find the sum of any arithmetic series.

$$S = \frac{n(a + \ell)}{2}$$

Example 1 **Find the sum: 1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9**

$a = 1, \ell = 9, n = 9$, so $S = \frac{9(1 + 9)}{2} = \frac{9 \cdot 10}{2} = 45$

Example 2 **Find the sum: −9 + (−5) + (−1) + 3 + 7 + 11 + 15**

$a = 29, \ell = 15, n = 7$, so $S = \frac{7(-9 + 15)}{2} = \frac{7 \cdot 6}{2} = 21$

Find the sum of each arithmetic series.

1. 3 + 6 + 9 + 12 + 15 + 18 + 21 + 24 **108**

2. 10 + 15 + 20 + 25 + 30 + 35 + 40 + 45 + 50 **270**

3. −21 + (−16) + (−11) + (−6) + (−1) + 4 + 9 + 14 **−28**

4. even whole numbers from 2 through 100 **2550**

5. odd whole numbers between 0 and 100 **2500**

NAME ______________________ DATE ____________ PERIOD _____

4-8 Study Guide and Intervention

Writing Equations from Patterns

Look for Patterns A very common problem-solving strategy is to **look for a pattern**. Arithmetic sequences follow a pattern, and other sequences can follow a pattern.

Example 1 **Find the next three terms in the sequence 3, 9, 27, 81, … .**

Study the pattern in the sequence.

$3 \xrightarrow{\times 3} 9 \xrightarrow{\times 3} 27 \xrightarrow{\times 3} 81$

Successive terms are found by multiplying the last given term by 3.

$81 \xrightarrow{\times 3} 243 \xrightarrow{\times 3} 729 \xrightarrow{\times 3} 2187$

The next three terms are 243, 729, 2187.

Example 2 **Find the next three terms in the sequence 10, 6, 11, 7, 12, 8, … .**

Study the pattern in the sequence.

$10 \xrightarrow{-4} 6 \xrightarrow{+5} 11 \xrightarrow{-4} 7 \xrightarrow{+5} 12 \xrightarrow{-4} 8$

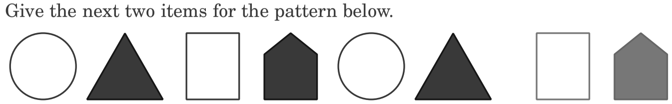
Assume that the pattern continues.

$8 \xrightarrow{+5} 13 \xrightarrow{-4} 9 \xrightarrow{+5} 14$

The next three terms are 13, 9, 14.

Exercises

1. Give the next two items for the pattern below.



Give the next three numbers in each sequence.

2. 2, 12, 72, 432, …
2592, 15,552, 93,312

3. 7, −14, 28, −56, …
112, −224, 448

4. 0, 10, 5, 15, 10, …
20, 15, 25

5. 0, 1, 3, 6, 10, …
15, 21, 28

6. $x - 1, x - 2, x - 3, \ldots$
$\mathbf{x - 4, x - 5, x - 6}$

7. $x, \frac{x}{2}, \frac{x}{3}, \frac{x}{4}, \ldots$
$\mathbf{\frac{x}{5}, \frac{x}{6}, \frac{x}{7}}$

Lesson 4-8

NAME ______________________ DATE ____________ PERIOD _____

4-8 Study Guide and Intervention *(continued)*

Writing Equations from Patterns

Write Equations Sometimes a pattern can lead to a general rule that can be written as an equation.

Example **Suppose you purchased a number of packages of blank compact disks. If each package contains 3 compact disks, you could make a chart to show the relationship between the number of packages of compact disks and the number of disks purchased. Use x for the number of packages and y for the number of compact disks.**

Make a table of ordered pairs for several points of the graph.

Number of Packages	1	2	3	4	5
Number of CDs	3	6	9	12	15

The difference in the x values is 1, and the difference in the y values is 3. This pattern shows that y is always three times x. This suggests the relation $y = 3x$. Since the relation is also a function, we can write the equation in functional notation as $f(x) = 3x$.

Exercises

1. Write an equation for the function in functional notation. Then complete the table.

x	−1	0	1	2	3	4
y	−2	2	6	**10**	**14**	**18**

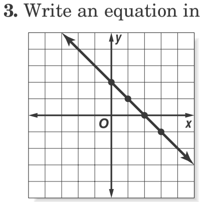
$\mathbf{f(x) = 4x + 2}$

2. Write an equation for the function in functional notation. Then complete the table.

x	−2	−1	0	1	2	3
y	10	7	4	**1**	**−2**	**−5**

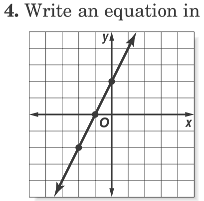
$\mathbf{f(x) = -3x + 4}$

3. Write an equation in functional notation.



$\mathbf{f(x) = -x + 2}$

4. Write an equation in functional notation.



$\mathbf{f(x) = 2x + 2}$

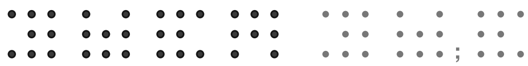
Answers

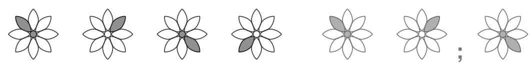
NAME ______________________ DATE ________ PERIOD _____

4-8 Skills Practice

Writing Equations from Patterns

Find the next two items for each pattern. Then find the 19th figure in the pattern.

1. 

2. 

Find the next three terms in each sequence.

3. 1, 4, 10, 19, 31, … **46, 64, 85**

4. 15, 14, 16, 15, 17, 16, … **18, 17, 19**

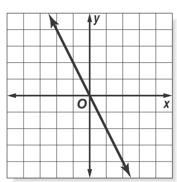
5. 29, 28, 26, 23, 19, … **14, 8, 1**

6. 2, 3, 2, 4, 2, 5, … **2, 6, 2**

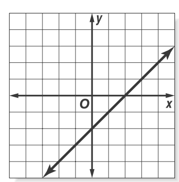
7. $x, x-1, x-2, \ldots$ **$x-3$, $x-4$, $x-5$**

8. $y, 4y, 9y, 16y, \ldots$ **$25y$, $36y$, $49y$**

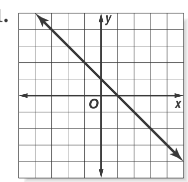
Write an equation in function notation for each relation.

9. 

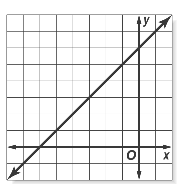
$f(x) = -2x$

10. 

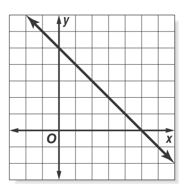
$f(x) = x - 2$

11. 

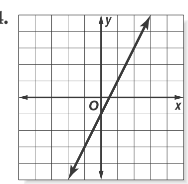
$f(x) = 1 - x$

12. 

$f(x) = x + 6$

13. 

$f(x) = 5 - x$

14. 

$f(x) = 2x - 1$

Lesson 4-8

NAME ______________________ DATE ________ PERIOD _____

4-8 Practice (Average)

Writing Equations from Patterns

1. Give the next two items for the pattern. Then find the 21st figure in the pattern.

Find the next three terms in each sequence

2. $-5, -2, -3, 0, -1, 2, 1, 4, \ldots$ **3, 6, 5**

3. 0, 1, 3, 6, 10, 15, … **21, 28, 36**

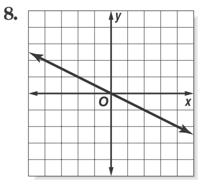
4. 0, 1, 8, 27, … **64, 125, 216**

5. 3, 2, 4, 3, 5, 4, … **6, 5, 7**

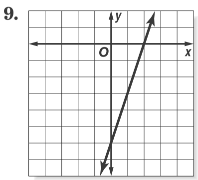
6. $a + 1, a + 4, a + 9, \ldots$
$a + 16$, $a + 25$, $a + 36$

7. $3d - 1, 4d - 2, 5d - 3, \ldots$
$6d - 4$, $7d - 5$, $8d - 6$

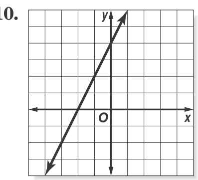
Write an equation in function notation for each relation.

8. 

$f(x) = -\frac{1}{2}x$

9. 

$f(x) = 3x - 6$

10. 

$f(x) = 2x + 4$

BIOLOGY For Exercises 11 and 12, use the following information.

Male fireflies flash in various patterns to signal location and perhaps to ward off predators. Different species of fireflies have different flash characteristics, such as the intensity of the flash, its rate, and its shape. The table below shows the rate at which a male firefly is flashing.

Time (seconds)	1	2	3	4	5
Number of Flashes	2	4	6	8	10

11. Write an equation in function notation for the relation. **$f(t) = 2t$, where t is the time in seconds and $f(t)$ is the number of flashes**

12. How many times will the firefly flash in 20 seconds? **40**

13. **GEOMETRY** The table shows the number of diagonals that can be drawn from one vertex in a polygon. Write an equation in function notation for the relation and find the number of diagonals that can be drawn from one vertex in a 12-sided polygon. **$f(s) = s - 3$, where s is the number of sides and $f(s)$ is the number of diagonals; 9**

Sides	3	4	5	6
Diagonals	0	1	2	3

NAME ______________________ DATE ________ PERIOD _____

4-8 Reading to Learn Mathematics

Writing Equations From Patterns

Pre-Activity **Why is writing equations from patterns important in science?**

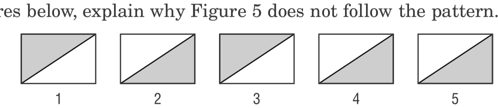
Read the introduction to Lesson 4-8 at the top of page 240 in your textbook.

- What is meant by the term *linear pattern*? **The points line up along a straight line.**
- Describe any arithmetic sequences in the data. **Both the volume of water and the volume of ice are arithmetic sequences. The volume of water has a common difference of 11 and the volume of ice has a common difference of 12.**

Reading the Lesson

1. What is meant by the term *inductive reasoning*? **making a conclusion based on a pattern of examples**

2. For the figures below, explain why Figure 5 does not follow the pattern.



If you follow the pattern of shading the top triangle, then the bottom, then the top, and so on, Figure 5 should have the top triangle shaded.

3. Describe the steps you would use to find the pattern in the sequence 1, 5, 25, 125, … . **Subtract successive terms to find out if it is an arithmetic sequence. It is not. So study the successive terms to see if multiplication or division is used. The pattern is to multiply each term by 5 to get the next term.**

Helping You Remember

4. What are some basic things to remember when you are trying to discover whether there is a pattern in a sequence of numbers? **Sample answer: Look to see whether successive terms increase or decrease by the same amount. If not, look to see whether successive terms have been multiplied or divided by the same number.**

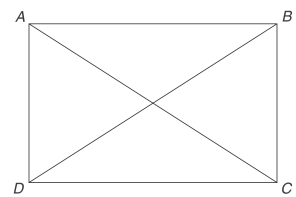
Lesson 4-8

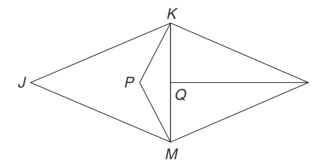
NAME ______________________ DATE ________ PERIOD _____

4-8 Enrichment

Traceable Figures

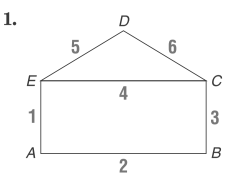
Try to trace over each of the figures below without tracing the same segment twice.



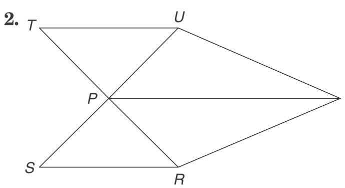


The figure at the left cannot be traced, but the one at the right can. The rule is that a figure is traceable if it has no points, or exactly two points where an odd number of segments meet. The figure at the left has three segments meeting at each of the four corners. However, the figure at the right has exactly two points, L and Q, where an odd number of segments meet.

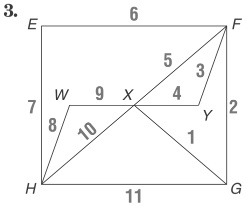
Determine whether each figure can be traced. If it can, then name the starting point and number the sides in the order in which they should be traced.

1. 

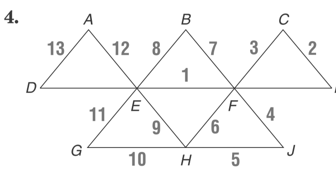
Yes; *E*

2. 

No

3. 

Yes; *X*

4. 

Yes; *D*

Answers